

Novel interferometer to beat the standard quantum limit using optical transverse modes in multimode waveguide

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Abstract

We propose a novel interferometer by using optical transverse modes in multimode waveguide that can beat the standard quantum limit. In the scheme, the classical simulation of N -particle quantum entangled states is generated by using N independent classical fields and linear optical elements. Similar to the quantum-enhanced measurements, the classical simulation can also achieve \sqrt{N} enhancement over the precision of the measurement N times for independent fields. Due to only using classical fields and linear optical elements, the scheme can be realized much more easily.

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Optical interferometric techniques are in particular widely used in ultimate sensitivity measurements, such as gravity-wave detection [1], nanometric displacement measurement [2] and optical gyroscopes [3]. In these measurements, the ultimate sensitivity is conventionally bounded by the quantum nature of the electromagnetic field. It has been shown that the so-called shot noise or standard quantum limit are due to the vacuum fluctuations coupled to the interferometer and to the random motion of the mirrors induced by the radiation pressure fluctuations [4]. However, these conventional limits are not as fundamental as the Heisenberg limits [5], and can be beaten by using quantum entanglement [6] and squeezing [7]. Quantum entanglement has been proved to allow a precision enhancement equal to the square root of the number N of employed particles, which can achieve the Heisenberg limits [6]. But there is an enormous difficulty in the quantum-enhanced measurement, which is usually very complicated to realize multi-particle quantum entanglement even as few as 5 or 6 particles [8].

Recently, “mode-entangled states” based on the transverse modes of classical optical fields propagating in multimode waveguides are proposed as classical simulation of quantum entangled states [9]. It is interesting that the mode-entangled states can also exhibit the nonlocal correlations, such as the violation of Bell’s inequality. The states can be regarded as the nonlocal generalization of the transient interference effect between two independent laser beams [10] and explained by a random phase ensemble model based on classical electromagnetics [11]. The simulation not only helps to understand the nonlocal properties of quantum entanglement from a new viewpoint, but also arouses interest in a full optical quantum computation scheme based on the transverse modes of classical fields [12, 13].

In this letter, we will propose a novel interferometer to beat the standard quantum limit using mode-entangled states. Similar to the quantum-enhanced measurements, the N -field mode-entangled states can also achieve \sqrt{N} enhancement over the precision of the measurement N times for independent fields. Moreover, the interferometer can be realized more easily than the quantum-enhanced measurement due to only using classical fields and linear optical elements. Before going into the scheme of the new interferometer, we would like to introduce an ordinary interferometer using optical transverse modes in multimode waveguide.

Considering a weakly guiding, symmetric slab waveguide, an optical field in the propagation z direction is restricted within the core region, which has the higher refractive index

(RI) compared with that of the cladding. We assume that a dual-mode waveguide support two normal modes, namely TE_0 mode and TE_1 mode. Thus the coherent superposition state of the two modes can be described as

$$|\psi\rangle = \begin{pmatrix} C_0 e^{i\beta_0 z} \\ C_1 e^{i\beta_1 z} \end{pmatrix}, \quad (1)$$

where β_0 and β_1 are the propagation constants of the modes TE_0 and TE_1 , respectively. Apparently, a mode analyzer (MA) that contains a variable phase modulator θ and Y splitter proposed in [9] is a simple dual-mode waveguide interferometer. We can define an intensity difference operator for the MA's outputs,

$$\hat{A}(\theta) = \begin{pmatrix} 0 & e^{i\theta} \\ e^{-i\theta} & 0 \end{pmatrix}. \quad (2)$$

When the input field of MA is prepared in the mode superposition $|\psi\rangle$, the intensity difference of the MA's outputs can be obtained

$$\begin{aligned} A(\theta) &= \langle\psi| \hat{A}(\theta) |\psi\rangle \\ &= C_0 C_1^* e^{i(\Delta\beta z + \theta)} + C_1 C_0^* e^{-i(\Delta\beta z + \theta)}, \end{aligned} \quad (3)$$

with $\Delta\beta = \beta_1 - \beta_0$. When $C_0 = C_1 = 1/\sqrt{2}$, $A(\theta) = \cos(\Delta\beta z + \theta)$. By using error propagation theory and Eq. (3), it is very easy to evaluate an overall phase error $\Delta\theta = 1/\sqrt{N}$ for repeating the experiment N times, which is the standard quantum limit for the interferometers.

In the quantum-enhanced measurements, N -particle quantum entangled states are required to achieve \sqrt{N} precision enhancement. Similarly, in our scheme, N -field mode-entangled states are required. By using numerical simulation, the CNOT gate scheme for generating mode-entangled states has proved feasible in Ref. [11]. Here we propose a new scheme using only linear optical elements to realize the classical simulation of quantum entanglement. In the scheme, by using a properly designed directional coupler (DC), two independent classical fields prepared in mode superpositions are completely exchanged TE_0 mode or TE_1 mode. The two output fields of the DC can exhibit the violation of Bell's inequality in the correlation measurement scheme proposed in Ref. [9].

If classical fields are quasi-monochromatic (i.e., the spread Δk is much less than the midwave number k_0 , $\Delta k \ll k_0$), two fields are statistically independent, as mainly reflected

in their independent random phases ϕ_a, ϕ_b uniformly distributed in $[0, 2\pi]$. Then the states of two independent fields prepared in the mode superpositions are defined as

$$|\psi_a\rangle = e^{i\phi_a} \begin{pmatrix} C_0^a \\ C_1^a \end{pmatrix}, |\psi_b\rangle = e^{i\phi_b} \begin{pmatrix} C_0^b \\ C_1^b \end{pmatrix}, \quad (4)$$

where $C_{0,1}^a, C_{0,1}^b$ are the mode coefficients of two optical fields, respectively. By properly adjusting the coupling coefficient and length, we can design a dual-mode waveguide DC to realize mode separating or combining [12]. By using the finite differential beam propagating method (FD-BPM) [14], we have simulated numerically mode separating by using the properly designed DC. The result is shown in Fig. 1. When the two fields $|\psi_a\rangle, |\psi_b\rangle$ are respectively sent into two inputs of the DC to completely exchange their TE_1 modes, we obtain the output two fields that have become incoherent mode superpositions,

$$|\psi'_a\rangle = e^{i\phi_a} \begin{pmatrix} C_0^a \\ C_1^b e^{i\lambda} \end{pmatrix}, |\psi'_b\rangle = e^{i\phi_b} \begin{pmatrix} C_0^b \\ C_1^a e^{-i\lambda} \end{pmatrix}, \quad (5)$$

where $\lambda = \phi_b - \phi_a$ is the random phase difference uniformly distributed in $[0, 2\pi]$. The output state of the DC can be written as the product of the two incoherent mode superpositions,

$$|\psi'\rangle = |\psi'_a\rangle \otimes |\psi'_b\rangle = e^{i(\phi_a + \phi_b)} \begin{pmatrix} C_0^a C_0^b \\ e^{-i\lambda} C_0^a C_1^a \\ e^{i\lambda} C_1^b C_0^b \\ C_1^a C_1^b \end{pmatrix}. \quad (6)$$

Assumed $C_{0,1}^a, C_{0,1}^b = 1/\sqrt{2}$, the density matrix ρ describing the state is then obtained

$$\rho = |\psi'\rangle \langle \psi'| = \frac{1}{4} \begin{pmatrix} 1 & e^{i\lambda} & e^{-i\lambda} & 1 \\ e^{-i\lambda} & 1 & e^{-2i\lambda} & e^{-i\lambda} \\ e^{i\lambda} & e^{2i\lambda} & 1 & e^{i\lambda} \\ 1 & e^{i\lambda} & e^{-i\lambda} & 1 \end{pmatrix}. \quad (7)$$

Consider an ensemble of the independent and identical systems that are labeled by the random phase λ that satisfies normalization condition $\int_{\Lambda} \Phi(\lambda) d\lambda = 1$, where $\Phi(\lambda)$ is a distribution function and $\Lambda \subseteq [0, 2\pi]$ is spanned by λ . Due to $\int_{\Lambda} e^{i\lambda} \Phi(\lambda) d\lambda = 0$, the density matrix ρ can be reduced by the ensemble average of λ ,

$$\rho_{\lambda} = \frac{1}{4} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}. \quad (8)$$

Apparently, the density matrix can not be factorized into the production of separate density matrices and is also different from that of quantum entangled state. If properly chose a operator \hat{Q} , we can obtain the same expectation value of \hat{Q} as quantum entangled states. In the correlation measurement of Bell's inequality proposed in Ref. [9], the intensity difference operators of MAs are this kind of operators. Therefore, we can obtain the violation of Bell's inequality by using mode-entangled states. This imply that the inseparability of the states similar to quantum entanglement might be caused by a random phase mechanism. Here, by using the same correlation measurement, we can obtain the normalized correlation function,

$$\begin{aligned}
S(\theta_1, \theta_2) &= \frac{\langle \text{Tr} [\rho \hat{A}(\theta_1) \hat{B}(\theta_2)] \rangle_\lambda}{\sqrt{\langle \text{Tr} [\rho_a \hat{A}^2(\theta_1)] \rangle_\lambda} \sqrt{\langle \text{Tr} [\rho_b \hat{B}^2(\theta_2)] \rangle_\lambda}} \\
&= \frac{\int_\Lambda \cos(\theta_1 + \lambda) \cos(\theta_2 - \lambda) \Phi(\lambda) d\lambda}{\sqrt{\int_\Lambda \cos^2(\theta_1 + \lambda) \Phi(\lambda) d\lambda} \sqrt{\int_\Lambda \cos^2(\theta_2 - \lambda) \Phi(\lambda) d\lambda}} \\
&= \cos(\theta_1 + \theta_2),
\end{aligned} \tag{9}$$

where $\langle \dots \rangle_\lambda$ denote the ensemble averages $\int_\Lambda \dots \Phi(\lambda) d\lambda$, and $\hat{A}(\theta_1), \hat{B}(\theta_2)$ are the intensity difference operators of MAs operated on the fields $|\psi'_a\rangle$ and $|\psi'_b\rangle$, and the reduced density matrices ρ_a, ρ_b are the partial traces $\text{Tr}_b(\rho)$ and $\text{Tr}_a(\rho)$, respectively. Substituting the correlation function into the Bell inequality [15] (CHSH inequality [16]),

$$|B| = |S(\theta_1, \theta_2) - S(\theta_1, \theta'_2) + S(\theta'_1, \theta'_2) + S(\theta'_1, \theta_2)| \leq 2, \tag{10}$$

the violation can be obtained by proper choice of the phases θ_1 and θ_2 . By using FD-BPM and the method referred in Ref. [11], we numerically demonstrate the normalized correlation functions for the two fields, as shown in Fig. 2. And the maximum violations of Bell's inequality are obtained, as shown in Table 1, where the maximum values of $|B|$ are the average results of many λ 's sequences.

Similarly, we can obtain the classical simulation of 3-particle GHZ state [17] by using three independent classical fields and two DCs. First, two fields are exchanged their TE_1 modes by using the first DC, then one of the output fields with the third field are exchanged their TE_1 modes by using the second DC. The output fields can be obtained

$$|\psi'_a\rangle = e^{i\phi_a} \begin{pmatrix} C_0^a \\ C_1^b e^{i\lambda_1} \end{pmatrix}, |\psi'_b\rangle = e^{i\phi_b} \begin{pmatrix} C_0^b \\ C_1^c e^{i\lambda_2} \end{pmatrix}, |\psi'_c\rangle = e^{i\phi_c} \begin{pmatrix} C_0^c \\ C_1^a e^{i\lambda_3} \end{pmatrix}, \tag{11}$$

where $\lambda_1 = \phi_b - \phi_a, \lambda_2 = \phi_c - \phi_b, \lambda_3 = \phi_a - \phi_c$ are the phase differences, and ϕ_a, ϕ_b, ϕ_c uniformly distributed in $[0, 2\pi]$ are the random phases and $C_{0,1}^a, C_{0,1}^b, C_{0,1}^c$ are the mode coefficients of three optical fields, respectively. Then the three fields are sent to three separated MAs denoted by $\hat{A}(\theta_1), \hat{B}(\theta_2)$ and $\hat{C}(\theta_3)$ respectively, we obtain the correlation function for three intensity differences,

$$\begin{aligned}
S(\theta_1, \theta_2, \theta_3) &= \langle \hat{A}(\theta_1) \hat{B}(\theta_2) \hat{C}(\theta_3) \rangle \\
&= \iiint_{\Lambda} A(\theta_1, \lambda_1) B(\theta_2, \lambda_2) C(\theta_3, \lambda_3) \Phi_1(\lambda_1) \Phi_2(\lambda_2) \Phi_3(\lambda_3) d\lambda_1 d\lambda_2 d\lambda_3 \\
&= \iiint_{\Lambda} \cos(\theta_1 + \lambda_1) \cos(\theta_2 + \lambda_2) \cos(\theta_3 + \lambda_3) \Phi_1(\lambda_1) \Phi_2(\lambda_2) \Phi_3(\lambda_3) d\lambda_1 d\lambda_2 d\lambda_3 \\
&= \frac{1}{4} \cos(\theta_1 + \theta_2 + \theta_3),
\end{aligned} \tag{12}$$

where $\Phi_i(\lambda_i)$ are the distribution functions of λ_i . Obviously, the correlation function is similar to that of GHZ state except a normalization factor, that can also present Bell's theorem without inequalities.

By using the linear optical scheme to generate mode-entangled states, we propose a novel interferometer to beat the standard quantum limit, the scheme is shown as Fig. 3. In the scheme, N independent classical fields $|\psi_i\rangle$ ($i = 1 \dots N$) are prepared in mode superpositions. Then the N fields are completely exchanged their TE_1 modes by using $N - 1$ DCs. We obtain the output fields,

$$|\psi'_1\rangle = e^{i\phi_1} \begin{pmatrix} C_0^1 \\ C_1^2 e^{i\lambda_1} \end{pmatrix}, |\psi'_2\rangle = e^{i\phi_2} \begin{pmatrix} C_0^2 \\ C_1^3 e^{i\lambda_2} \end{pmatrix}, \dots, |\psi'_N\rangle = e^{i\phi_N} \begin{pmatrix} C_0^N \\ C_1^1 e^{i\lambda_N} \end{pmatrix}, \tag{13}$$

where $\lambda_i = \phi_{i+1} - \phi_i, \lambda_N = \phi_1 - \phi_N, (i = 1 \dots N - 1)$ are the phase differences, and $C_{0,1}^i, \phi_i$ are the mode coefficients and the random phases of the optical fields, respectively. Then the output fields are employed to measure a small phase difference θ , and the output states can be written as,

$$|\psi'_1\rangle = e^{i\phi_1} \begin{pmatrix} C_0^1 \\ C_1^2 e^{i(\lambda_1 + \theta)} \end{pmatrix}, |\psi'_2\rangle = e^{i\phi_2} \begin{pmatrix} C_0^2 \\ C_1^3 e^{i(\lambda_2 + \theta)} \end{pmatrix}, \dots, |\psi'_N\rangle = e^{i\phi_N} \begin{pmatrix} C_0^N \\ C_1^1 e^{i(\lambda_N + \theta)} \end{pmatrix}. \tag{14}$$

At last the fields are sent into the Y splitters, and the intensity differences are measured by photoelectric detectors. The detected photocurrents are passively subtracted and performed

correlation analysis. The correlation function of the intensity differences can be obtained

$$\begin{aligned}
S(\theta) &= \left\langle \hat{A}_1(\theta) \hat{A}_2(\theta) \dots \hat{A}_N(\theta) \right\rangle \\
&= \int_{\Lambda} \prod_{i=1}^N A_i(\theta, \lambda_i) \Phi_i(\lambda_i) d\lambda_i \\
&= \int_{\Lambda} \prod_{i=1}^N \cos(\theta + \lambda_i) \Phi_i(\lambda_i) d\lambda_i \\
&= \frac{1}{2^{N-1}} \cos(N\theta),
\end{aligned} \tag{15}$$

where $\Phi_i(\lambda_i)$ are the distribution functions of λ_i . The normalization factor $1/2^{N-1}$ can be removed by proper normalization procedure. As before, the correlation function $S(\theta)$ can be estimated with an error $\Delta^2 S(\theta) = [\cos(2N\theta) - \sin^2(N\theta)]/2^{2N-2}$. This means that the phase θ will have an error $\Delta\theta = \Delta S(\theta) / \left| \frac{\partial S(\theta)}{\partial \theta} \right| = 1/N$, that is the Heisenberg limit. This is a \sqrt{N} enhancement over the precision of N measurements on independent fields.

In order to avoid the influence of the independent photon number distributions of the independent fields, we can split one classical field into multiple beams, then modulate each beam by an independent random phase shift. And the multiple beams can be employed as multiple independent fields. Moreover the granularity of random phases hardly influences the measurement result [11], so that the random phases can be assigned with a finite number of discrete values uniformly distributed in $[0, 2\pi]$ to realize rapid phase ergodicity. In the scheme, the phase ergodicity might be one of the most important sources of imprecision.

In this letter, we have discussed a novel interferometer by using optical transverse modes in multimode waveguide that can beat the standard quantum limit. By using a new linear optical scheme, N -field mode-entangled states can be generated. Similar to the quantum-enhanced measurements, the N -field mode-entangled states have achieved \sqrt{N} enhancement over the precision of the measurement N times for independent fields. Compare to the quantum scheme, the scheme can be realized much more easily. Although, we have employed the interferometer using optical transverse modes, the results are generally applicable to other types of interferometers, in particular to the arrangements under development for gravity-wave detection.

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Fig. 1: BPM simulation result for the directional coupler to realize mode separating.

Fig. 2: The correlation functions $S(\theta_1, \theta_2)$ for mode-entangled states: (a) $|\Phi_1^+\rangle$, (b) $|\Phi_1^-\rangle$, (c) $|\Psi_1^+\rangle$, and (d) $|\Psi_1^-\rangle$.

Fig. 3: The scheme of the interferometer.

Tabel 1:

$ \Phi_1^+\rangle$	$\theta_1 = \frac{12}{46}\pi, \theta'_1 = \frac{73}{46}\pi, \theta_2 = \frac{66}{46}\pi, \theta'_2 = \frac{5}{46}\pi$	$\max B = 2.8174$
$ \Phi_1^-\rangle$	$\theta_1 = \frac{39}{46}\pi, \theta'_1 = \frac{19}{46}\pi, \theta_2 = \frac{24}{46}\pi, \theta'_2 = \frac{44}{46}\pi$	$\max B = 2.8222$
$ \Psi_1^+\rangle$	$\theta_1 = \frac{39}{46}\pi, \theta'_1 = \frac{100}{46}\pi, \theta_2 = \frac{88}{46}\pi, \theta'_2 = \frac{68}{46}\pi$	$\max B = 2.8152$
$ \Psi_1^-\rangle$	$\theta_1 = \frac{38}{46}\pi, \theta'_1 = \frac{18}{46}\pi, \theta_2 = \frac{87}{46}\pi, \theta'_2 = \frac{67}{46}\pi$	$\max B = 2.8218$

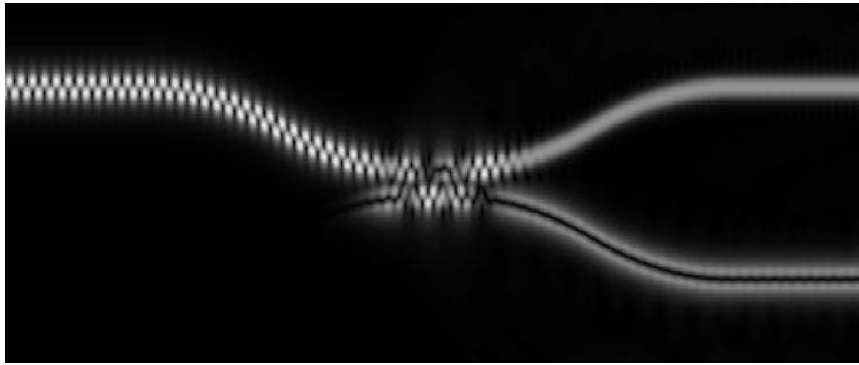


Fig. 1

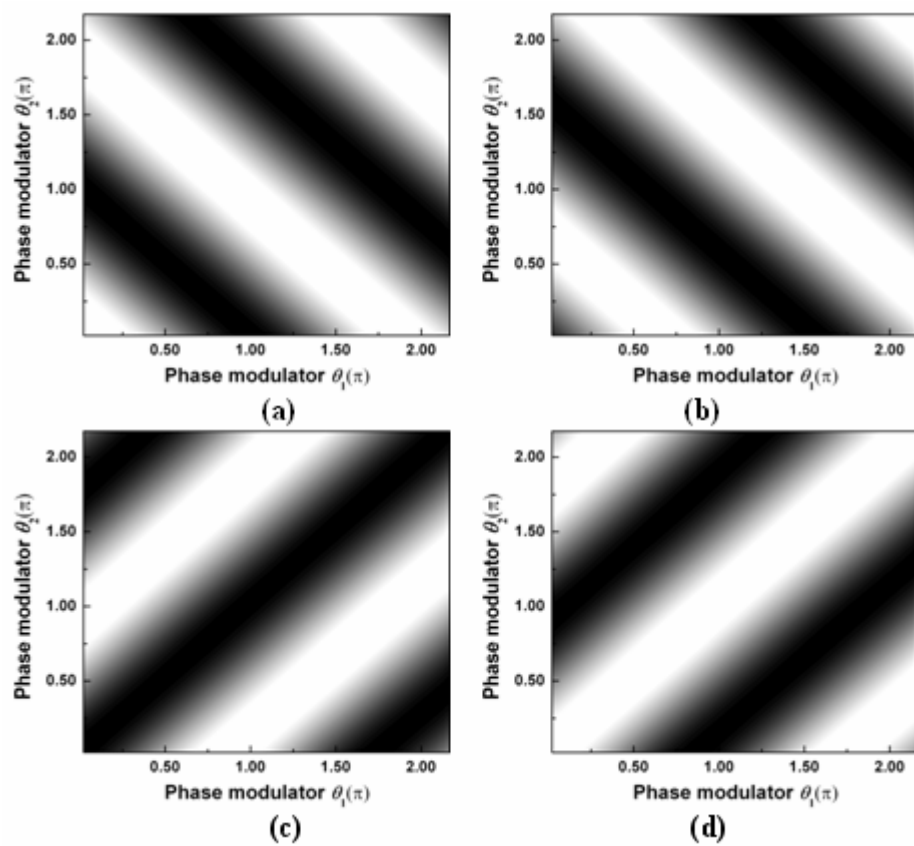


Fig. 2

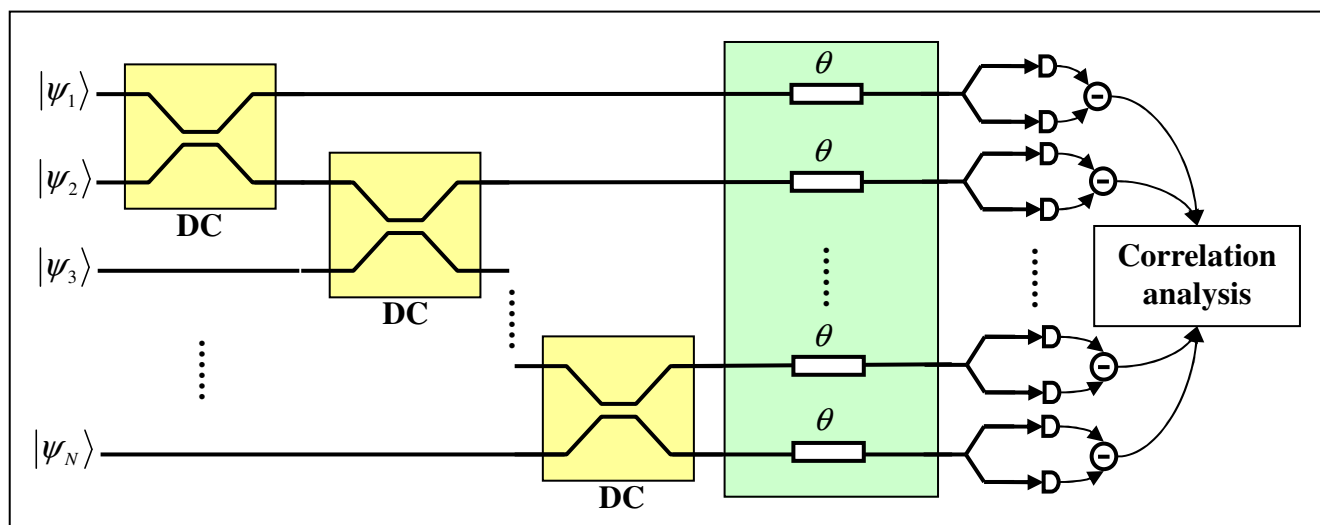


Fig. 3